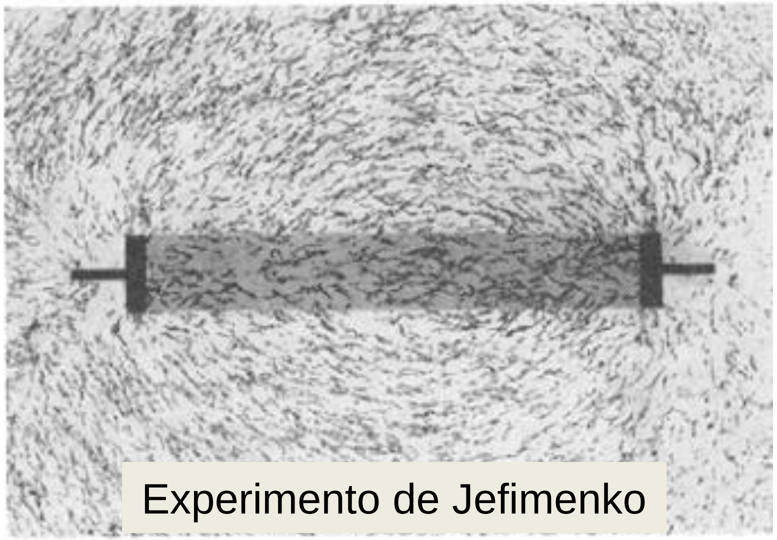
CARGAS SUPERFICIALES con QuickField



Experimento de Jefimenko

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CAMPUS MARE NOSTRUM

SOFTWARE 2D utilizado por los autores:

- EASY JAVA SIMULATION (EJS) → Diferencias Finitas → requiere programación elemental

  http://fem.um.es/Ejs/Ejs_es/index.html

  E. Martín, J.Muñoz, “Cargas superficiales y circuitos”, URSI, Tenerife, 2007.

- QUICK FIELD (QF) → Elementos Finitos → muy intuitivo para alumno.

Relevancia de la cargas superficiales

1) Determinan procesos físicos de interés:

   - Relajación de cargas en conductores
   - Conducción eléctrica

2) Nexo entre electrostática y circuitos eléctricos

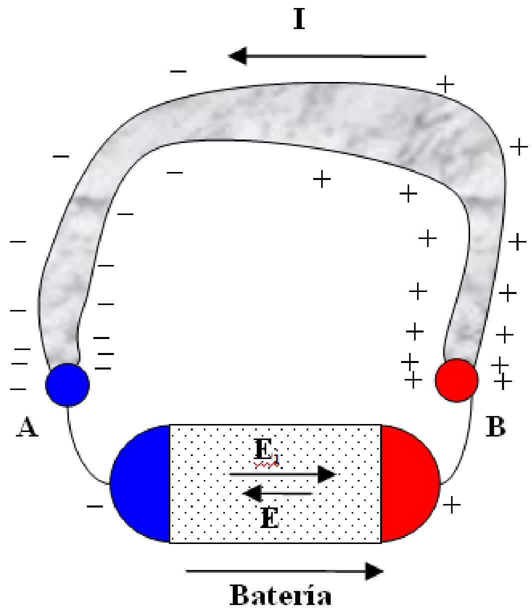
3) El papel de las cargas superficiales en conductores portadores de corriente eléctrica (Jefimenko, Sommerfeld, Jackson, Chabay-Sherwood) no aparece en los libros usuales de texto

Establecimiento de la corriente eléctrica en un circuito

- **E**: campo asociado a la distribución estática de cargas/conservativo/ se extiende al exterior de la batería
- **E_i**: bombea contracorriente/no conservativo/ localizado en el interior de la batería
- Corriente estacionaria: supone autoajuste (instantáneo) del campo E
- Distribución espacial no uniforme de cargas superficiales responsable de:

J.D. JACKSON:

- ❑ Fuentes del campo eléctrico en el exterior
- ❑ Mantienen el potencial a lo largo del conductor
- ❑ Fuentes locales del campo eléctrico en el interior
- ❑ Aseguran flujo confinado de la corriente eléctrica



LEYES IMPLICADAS

- Continuidad: $$\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$$ $$\frac{\partial\rho}{\partial t}=0\rightarrow\nabla\cdot\vec{J}=0$$

$$\rho(\vec{r},t)=\rho(\vec{r},0)\exp\left[\left(-\sigma/\varepsilon_0\right)t\right]\qquad \tau=\varepsilon_0/\sigma\approx 10^{-19}s$$

- Constitutivas: $$\vec{D}=\varepsilon\vec{E}$$ $$\vec{J}=\sigma\vec{E}$$ (L.Ohm)

- Ley de Gauss: $$D_n=Q_{\sup}\ \left(C/m^2\right)$$

$$\rho=\nabla\cdot\vec{D}=\nabla\cdot\left(\varepsilon\vec{E}\right)=\nabla\cdot\left(\varepsilon\vec{J}/\sigma\right)=\varepsilon/\sigma\,\nabla\cdot\vec{J}+\vec{J}\cdot\nabla\left(\varepsilon/\sigma\right)=\vec{J}\cdot\nabla\left(\varepsilon/\sigma\right)$$

(Si el material es homogéneo, no hay carga en volumen)

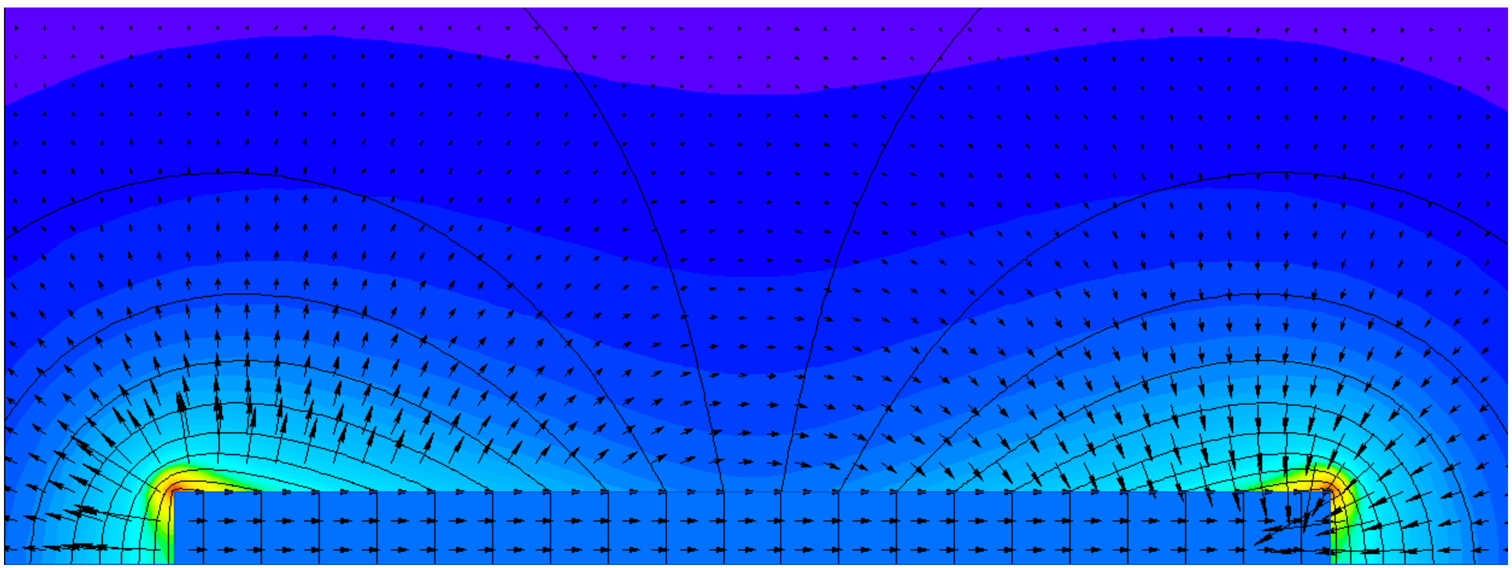
Caso 1: Conductor cilíndrico

Módulo: DC Conduction (axisymmetric)

$L = 4\ m,\ \Delta V = 2\ V$ ⟶ $E = \Delta V / L = 0.5\ V/m \quad (r < a)$

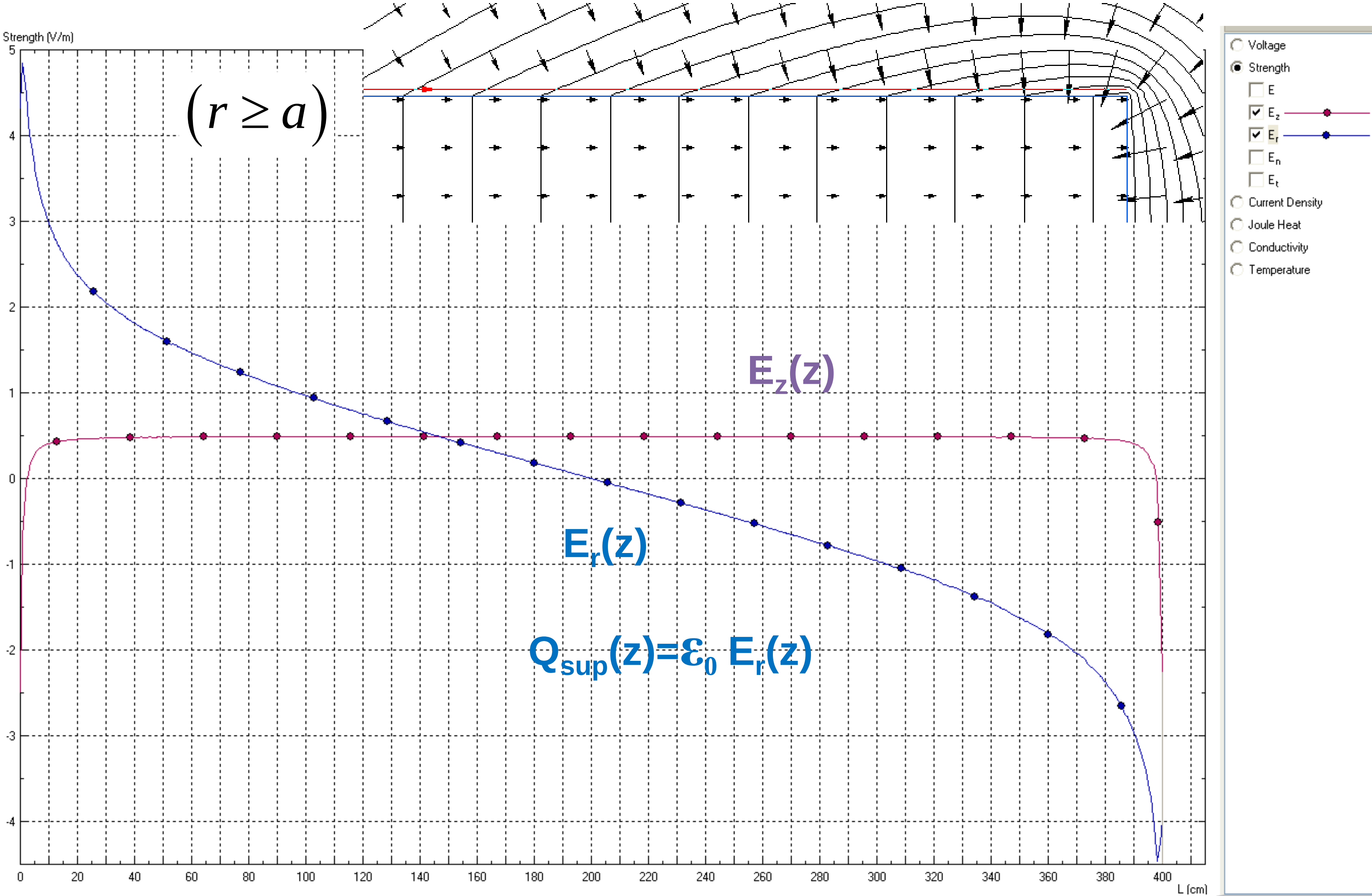
$\sigma = 100\ S/m$ ⟶ $J = \sigma E = 50\ A/m^2$, $\tau = \varepsilon_0 / \sigma \approx 10^{-13}\, s$

$a = 25\ cm$ ⟶ $I = J S = 9.8\ \mathrm{A}$, $R = \dfrac{1}{\sigma}\dfrac{L}{S} = 0.2\Omega$



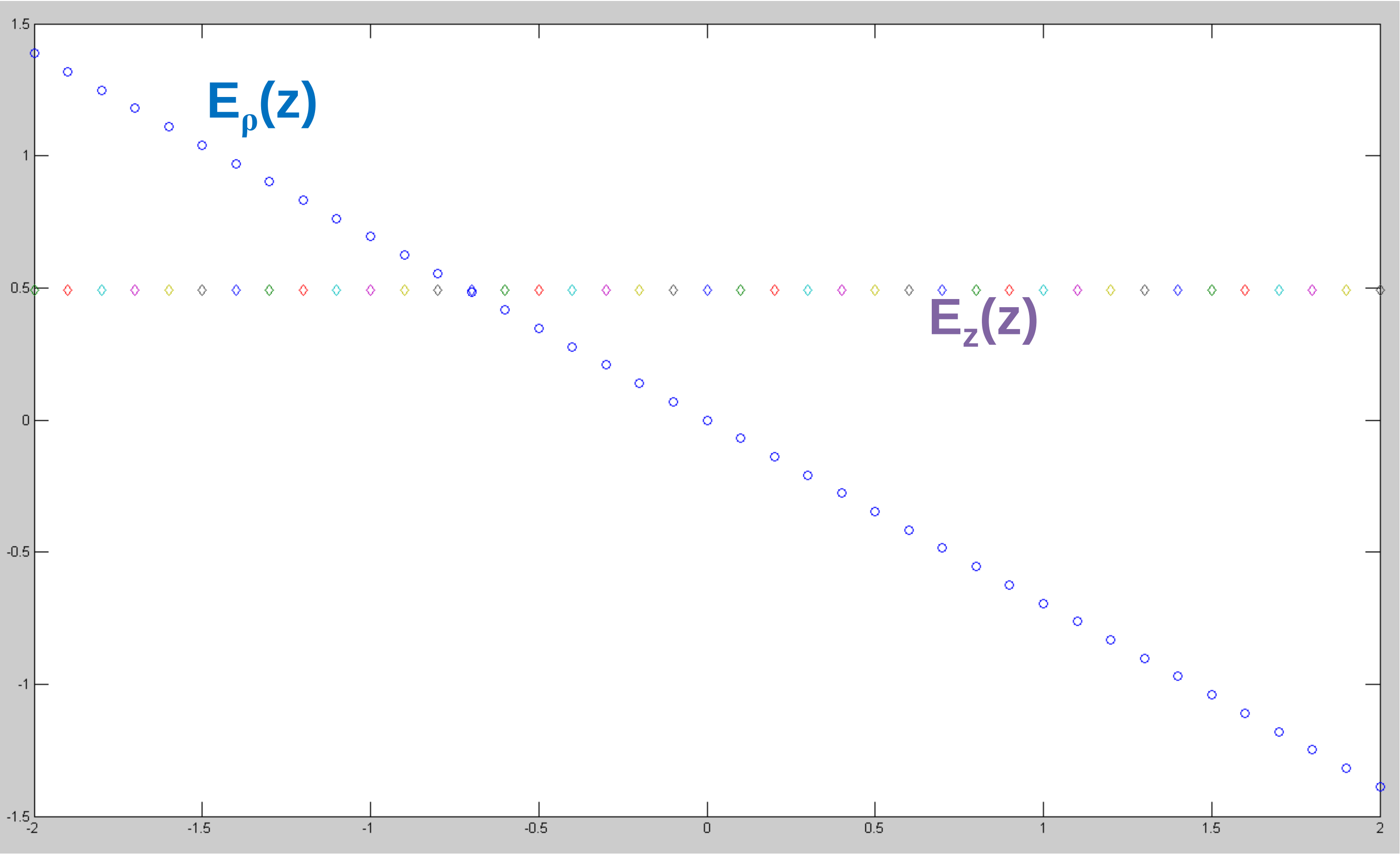
Distribución de campo y líneas equipotenciales

Contorno para obtención de Q_{sup}

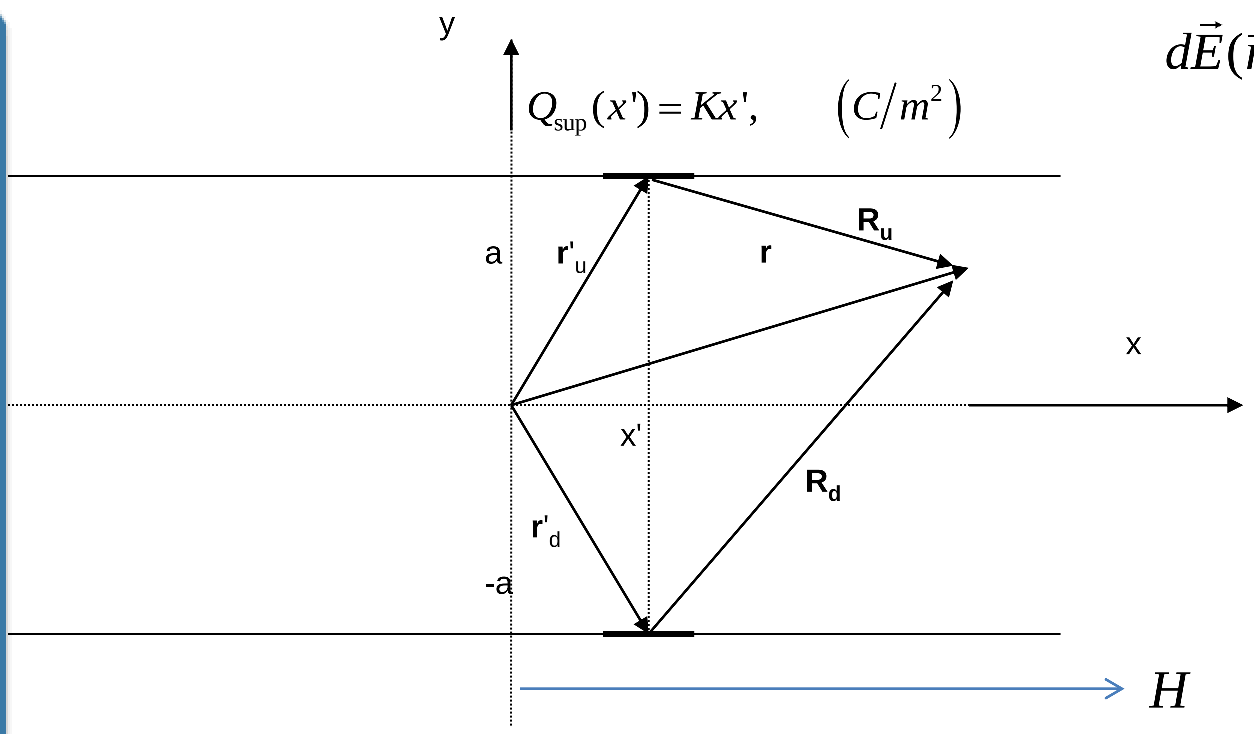


$$\vec{E} = \left[\frac{1}{\ln(\ell/a)}\left(\frac{RI + 2\phi_R}{2} - RI\frac{z}{\ell}\right)\frac{\hat{\rho}}{\rho} + \frac{RI}{\ell}\frac{\ln(\ell/\rho)}{\ln(\ell/a)}\hat{z}\right] \quad \text{if } \rho > a$$

(Assis-Hernandes)



ANÁLOGO ELECTROSTÁTICO: UN GRADIENTE LINEAL DE CARGA A LO LARGO DE UN SISTEMA-2D CON SIMETRÍA TRASLACIONAL PRODUCE UN CAMPO ELÉCTRICO UNIFORME (Russell)



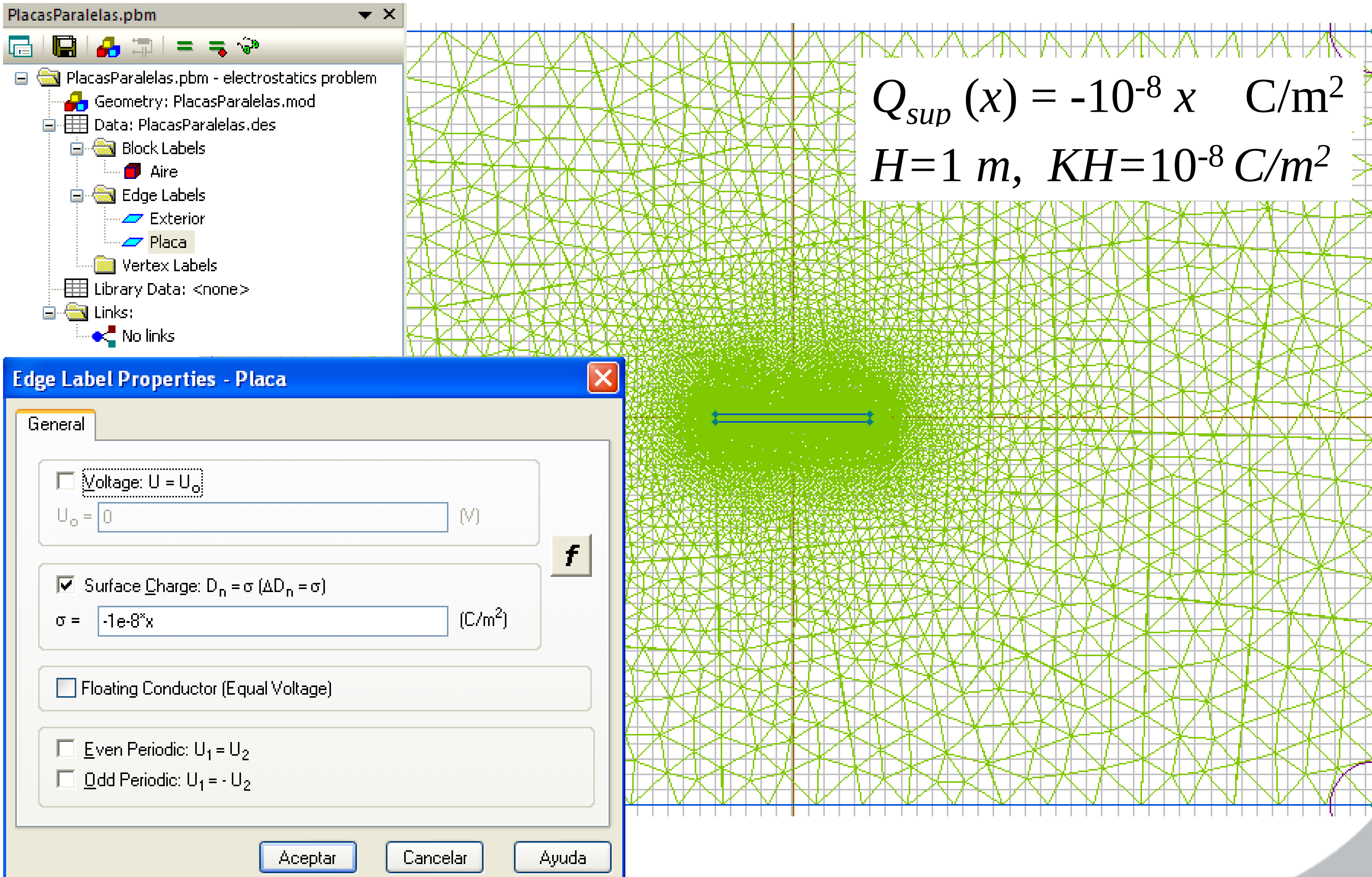
$$d\vec{E}(\vec{r}) = \frac{Q_{sup}(x')}{2\pi\varepsilon_0}\left(\frac{\vec{R}_u}{R_u^2} + \frac{\vec{R}_d}{R_d^2}\right)$$

$$(a, x << H)$$

$$E = E_x = -\frac{2KH}{\pi\varepsilon_0}$$

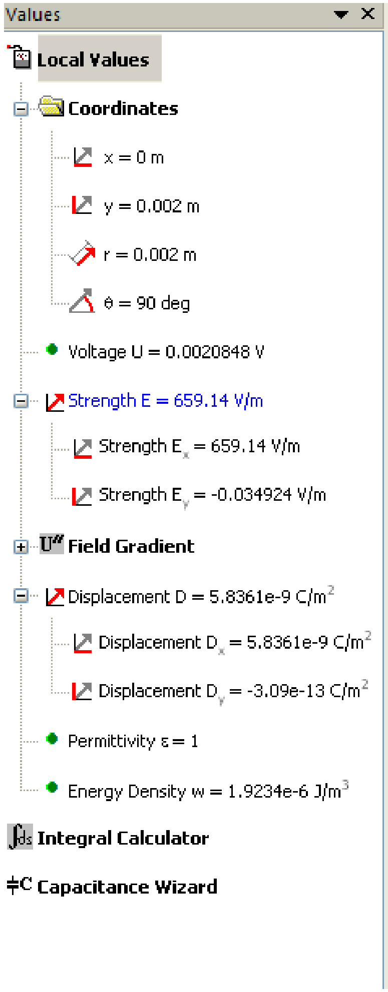
$$Q_{sup}(H) = KH$$

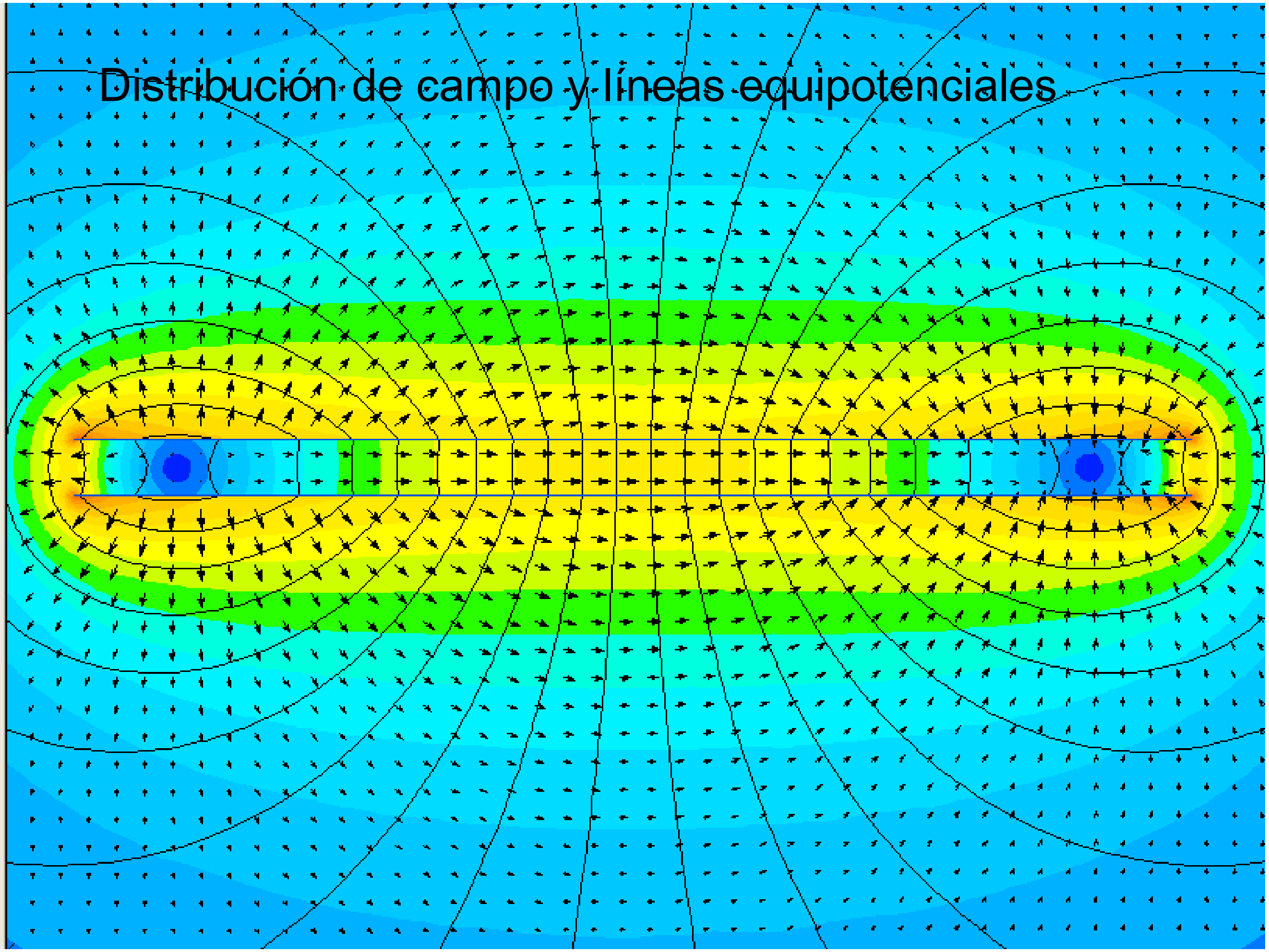
Caso 2: Placas Paralelas (Mod: Electrostatics)



$Q_{sup}(x) = -10^{-8}\,x \quad \text{C/m}^2$

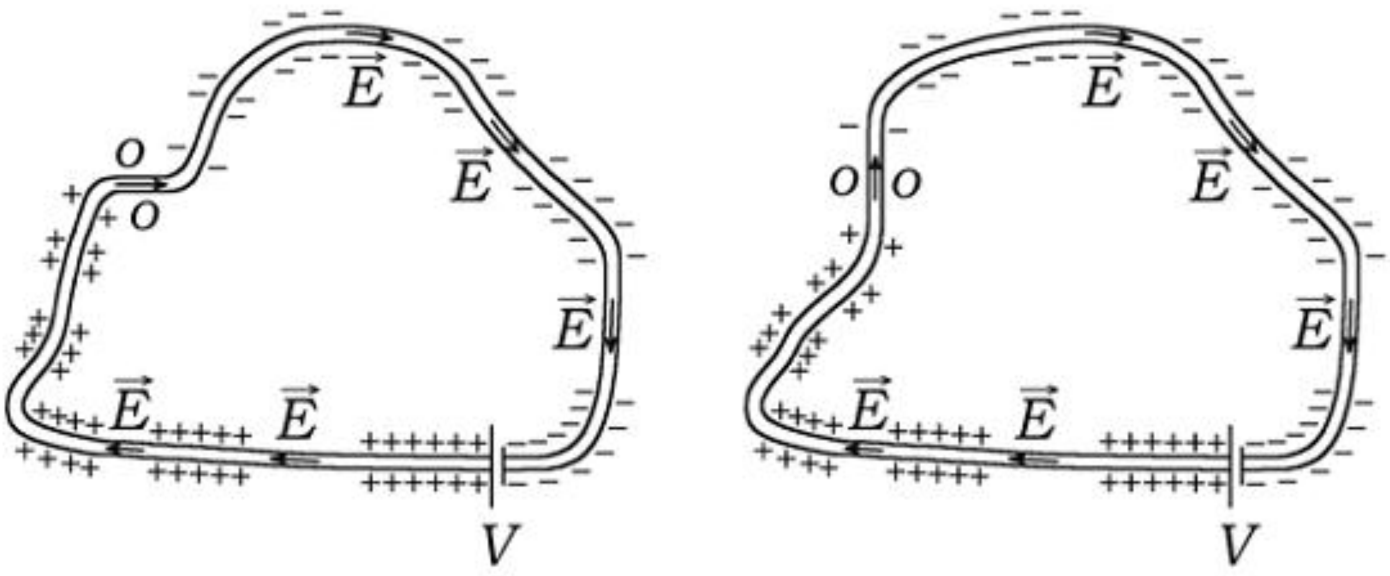
$H = 1\ m,\ \ KH = 10^{-8}\ C/m^2$





$$E_x\left(teórico\right) = 2(KH)/\pi\varepsilon_0 = 719 \text{ V/m} \qquad E_x\left(QF\right) = 659 \text{ V/m}$$

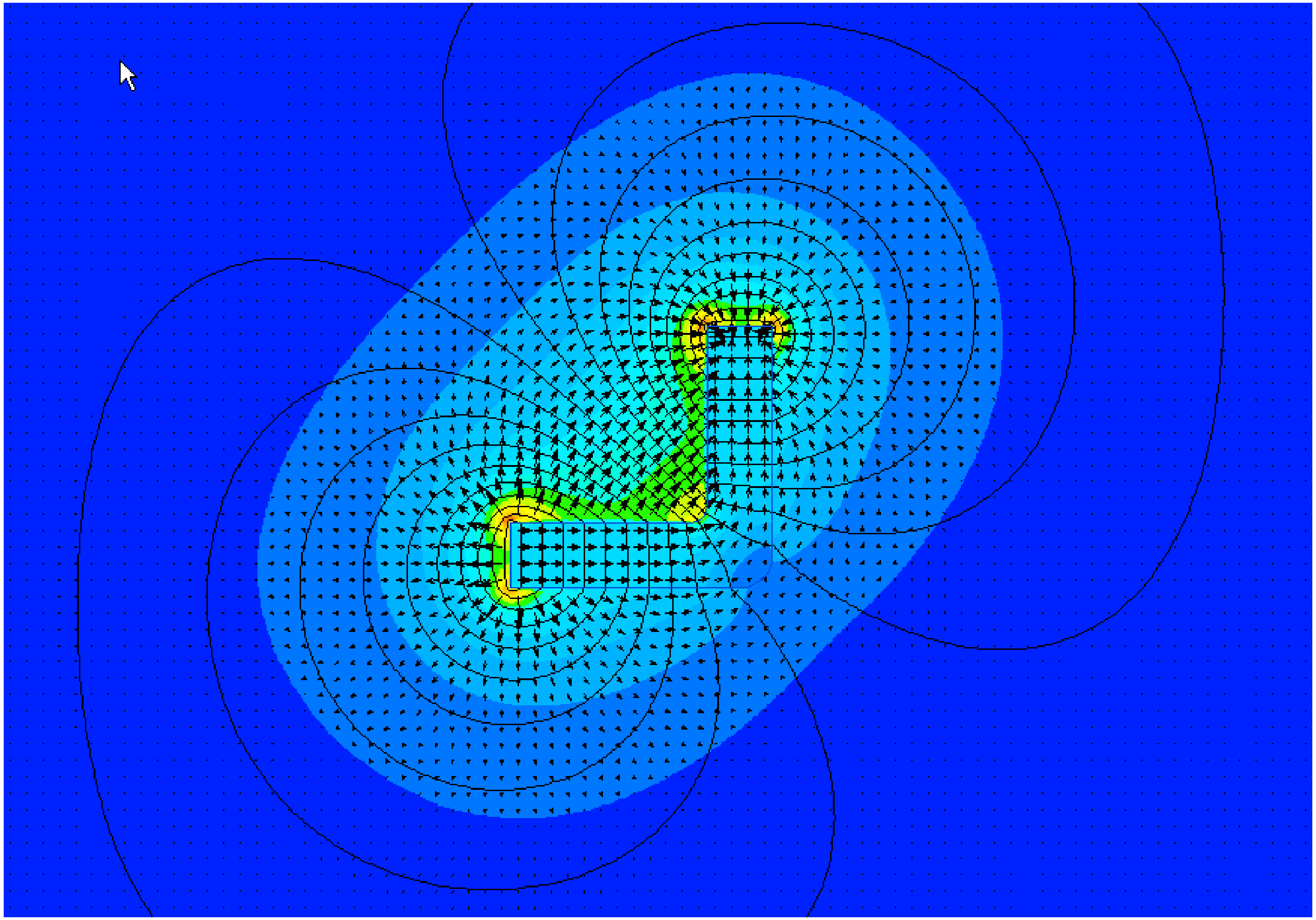
Caso 3: Conductor curvado



(Assis-Hernandes)

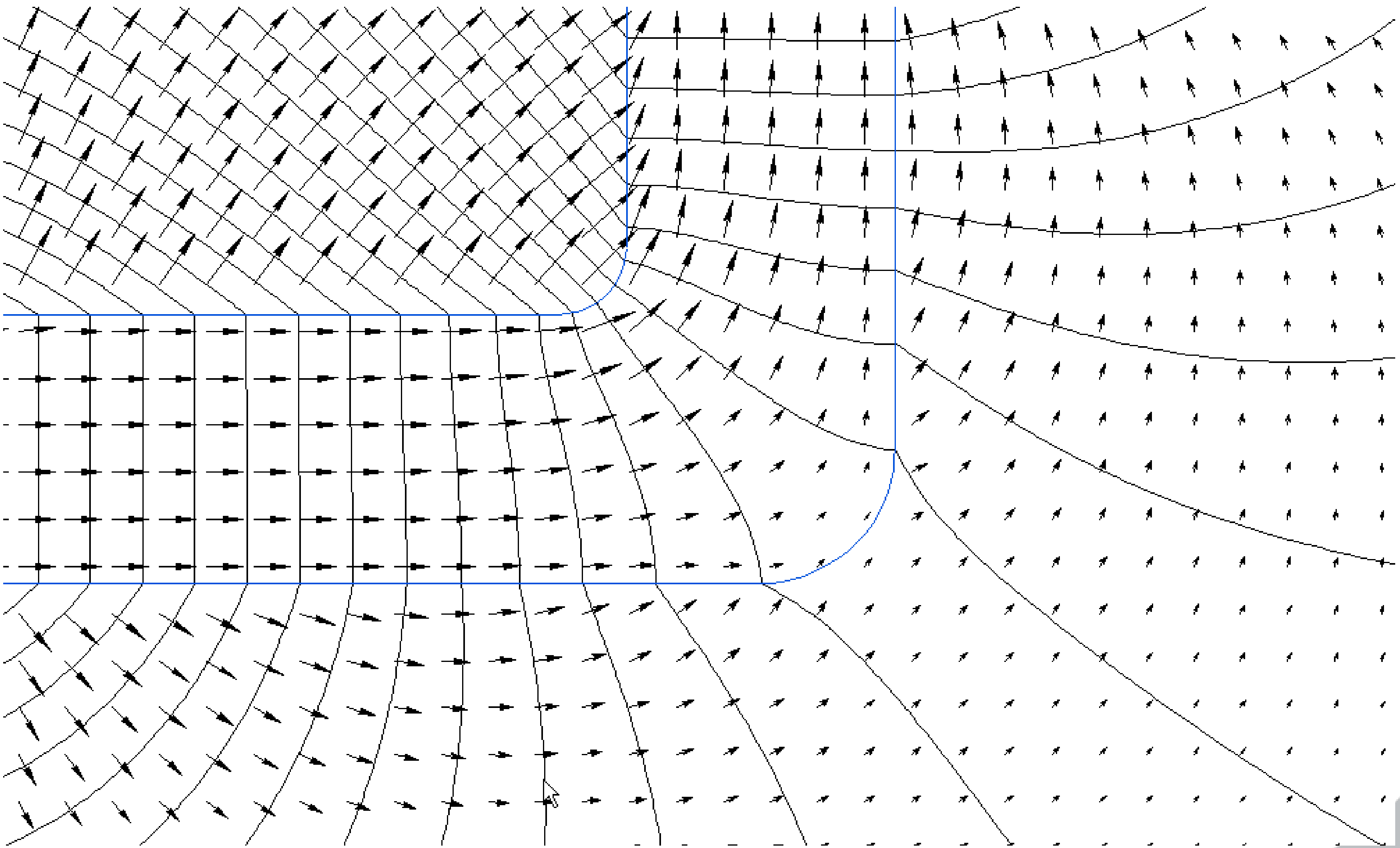
Módulo: Transient Electric

$\sigma = 10^{-11}\, S/m$, $\tau = \varepsilon_0/\sigma \approx 1s$

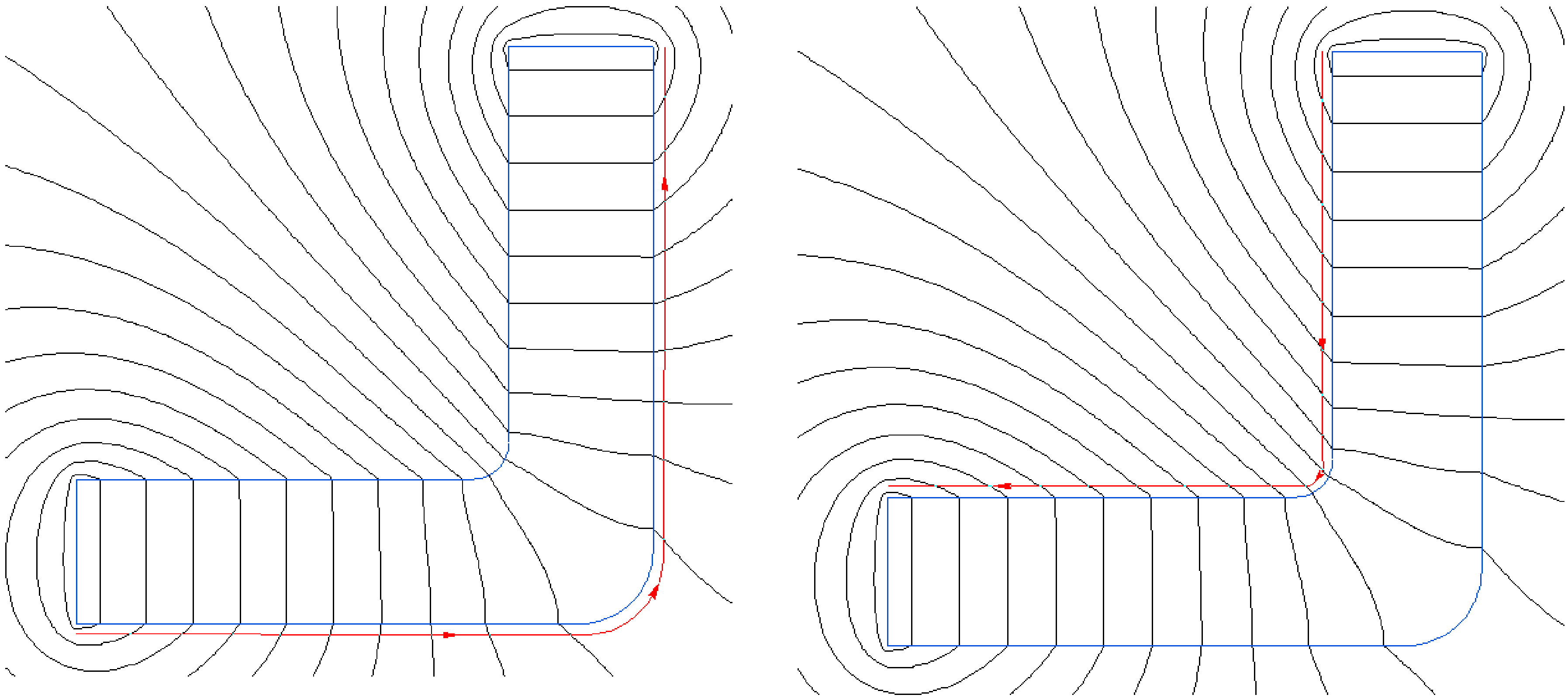


Distribución de campo y líneas equipotenciales

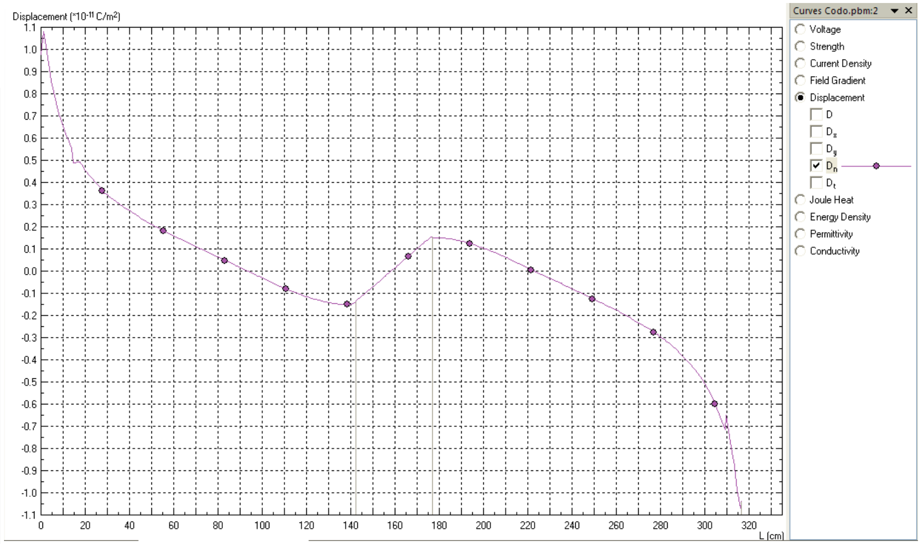
Cobre: $Q = \varepsilon_0 EA = \varepsilon_0 JA/\sigma = \varepsilon_0 I/\sigma = 1.5\text{x}10^{-19}\, I$ (C) (Rosser)

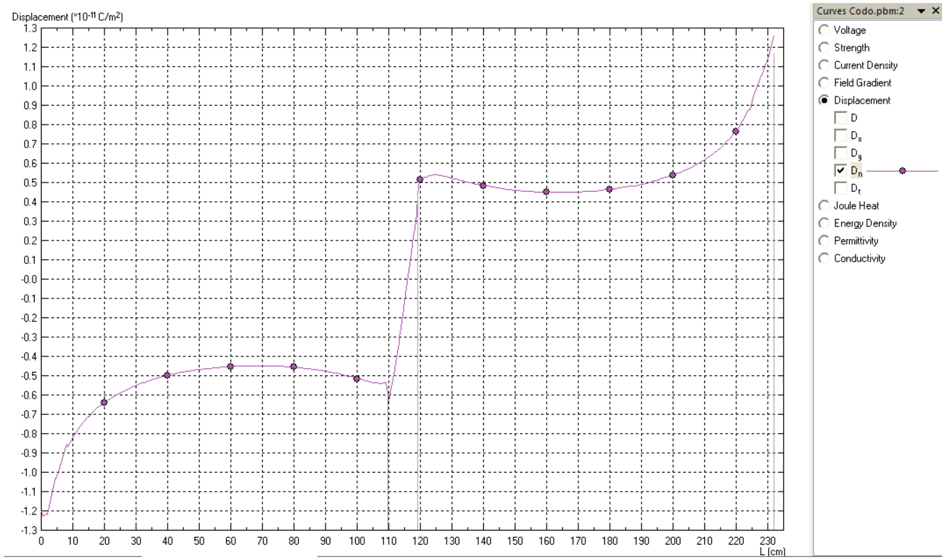


Contornos para la obtención de Q_{sup}



Q_{sup} (ext) $\rightarrow$ $\leftarrow$ Q_{sup} (int)



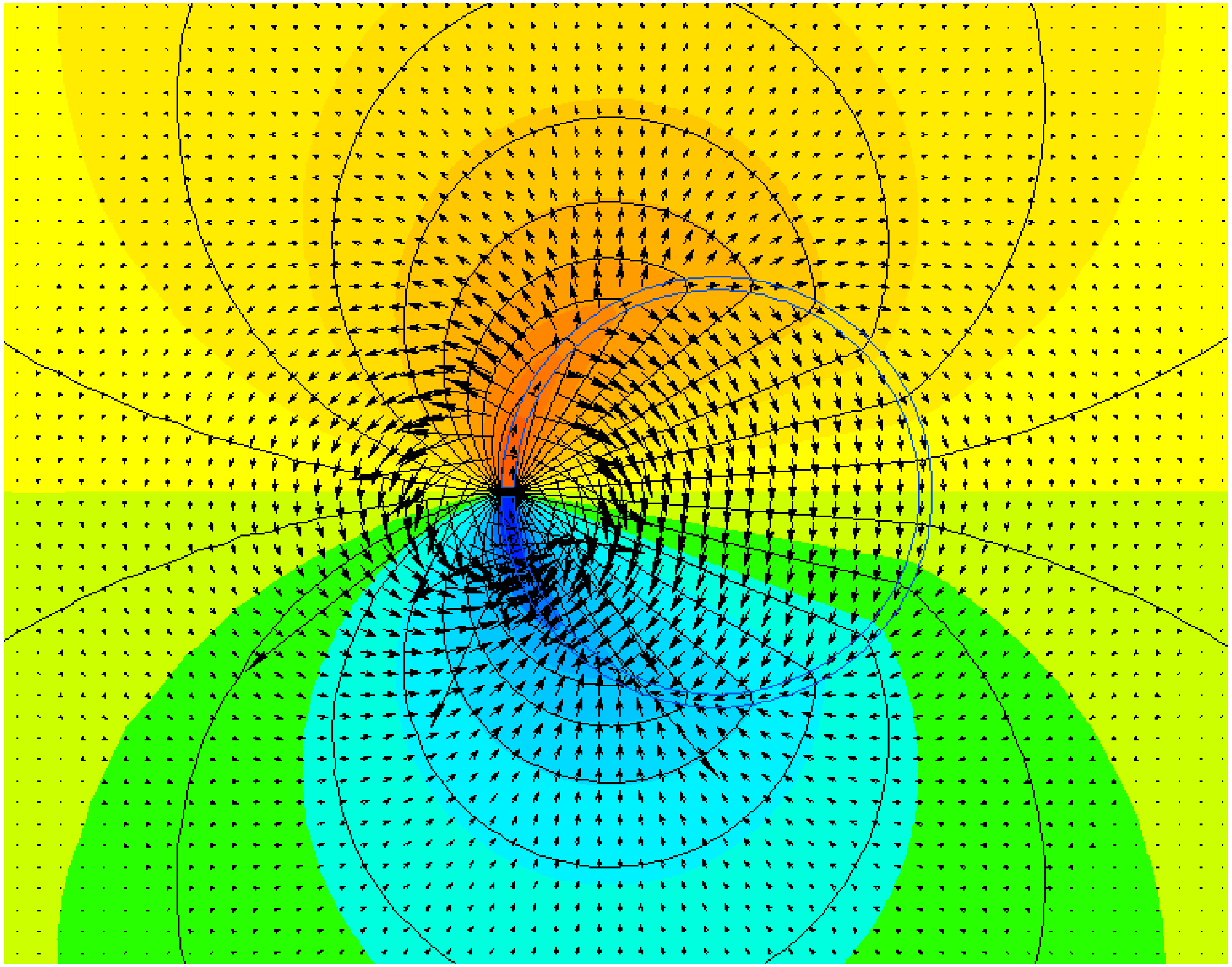


$\sigma = 10^{-11}\ S/m$ $\quad \Delta V = 1\ V.$ $\quad S = 0.4x1\ m^2 = 0.4\ m^2$

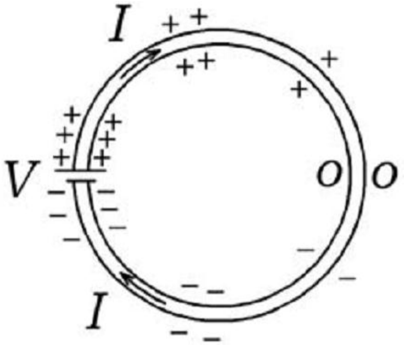
$Q = \varepsilon_0 ES = \varepsilon_0\, JS/\sigma = \varepsilon_0\, I/\sigma = 0.885\ I\ \ C.$

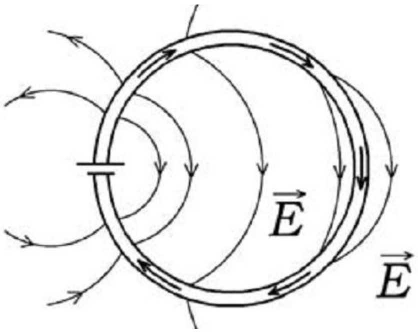
J_x (QF) = $3.9x10^{-12}\ A/m^2$ $\quad I = J_x S = 1.56\ x10^{-12}\ A.$

Caso 4: Conductor circular
Módulo: Transient Electric



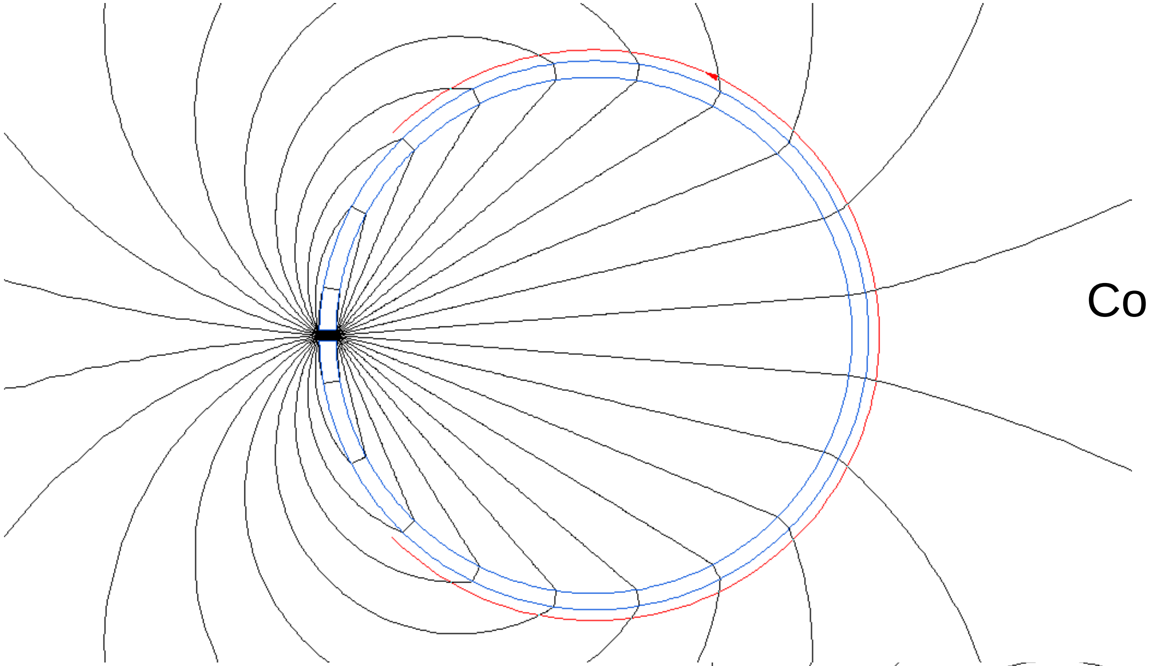
Mapa de potencial y vectores de campo



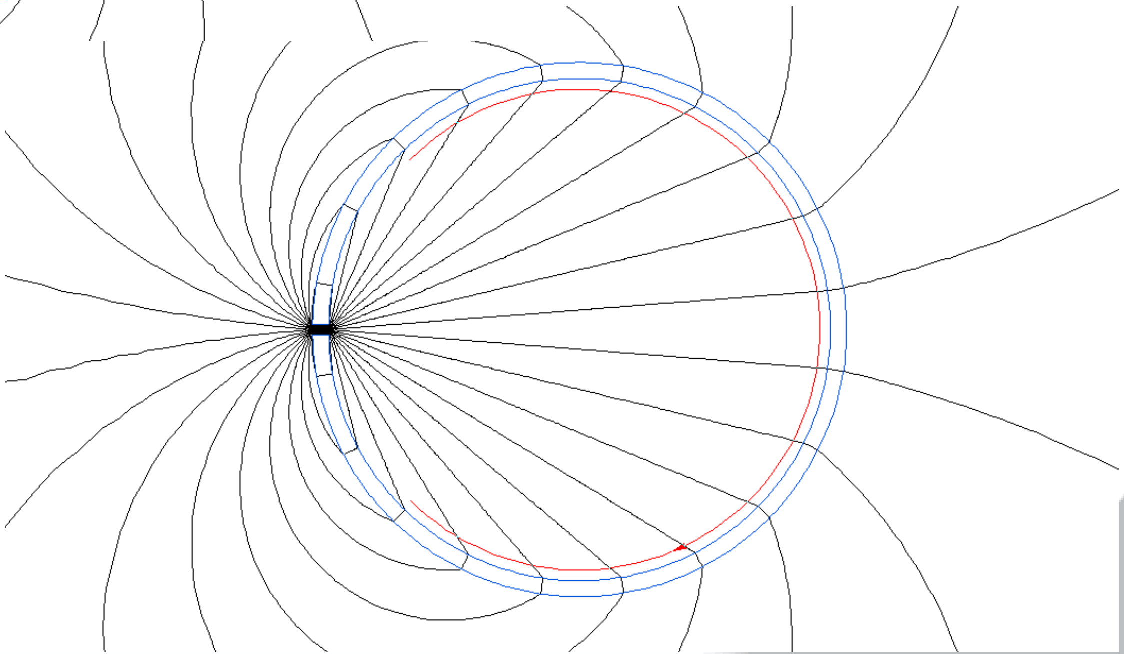


(Assis-Hernandes)

(M. A. Heald)

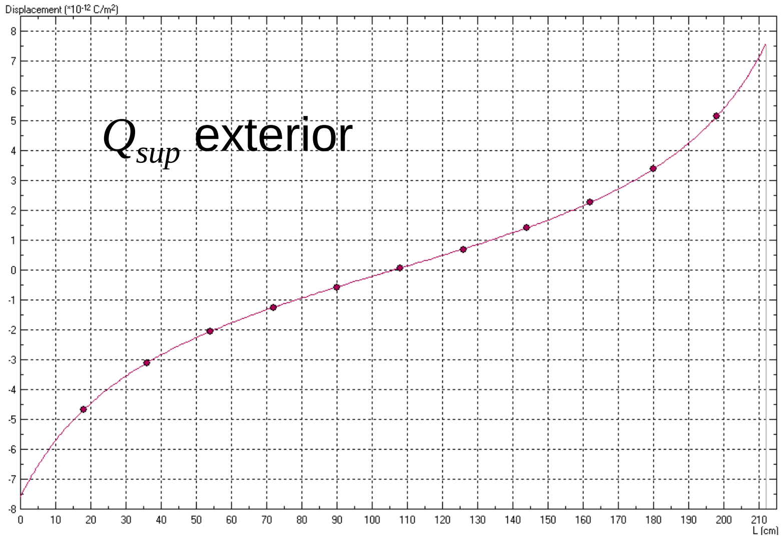


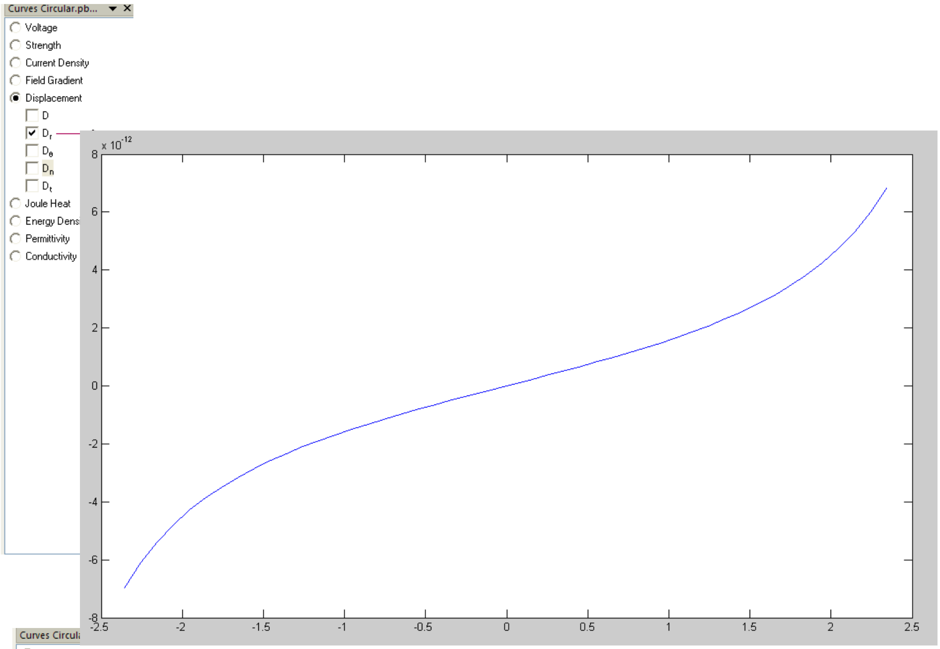
Contorno exterior Q_{sup}



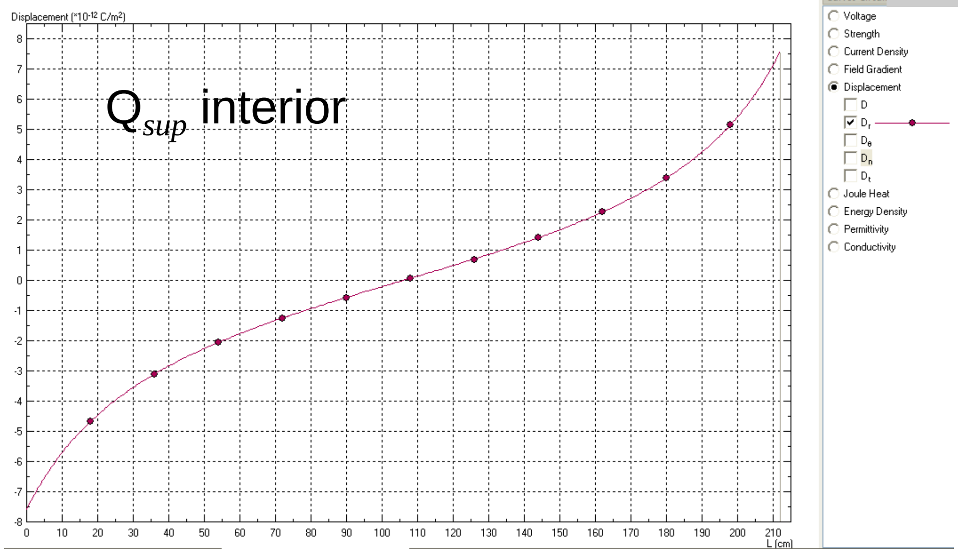
Contorno interior Q_{sup}

Q_{sup} exterior





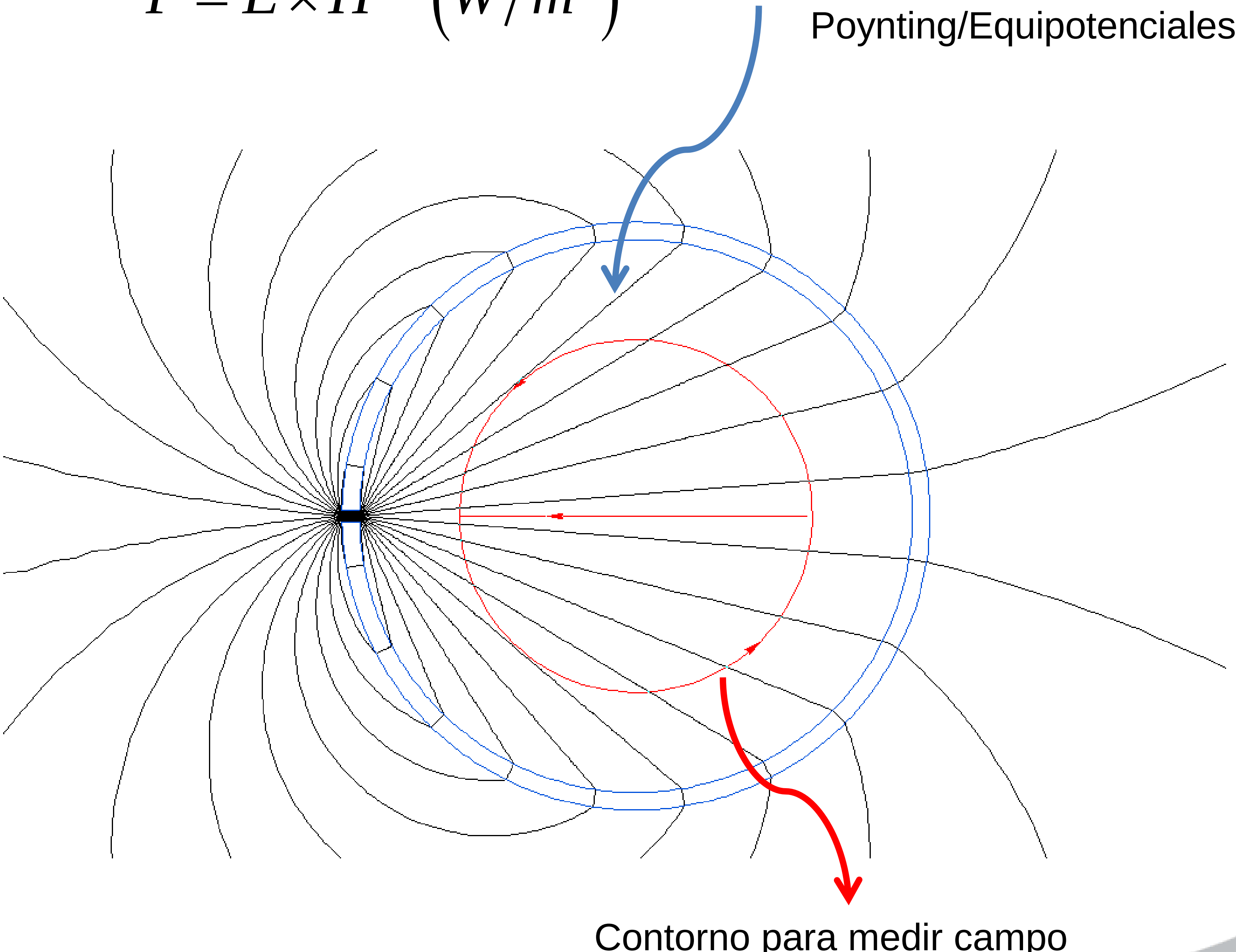
Q_{sup} interior



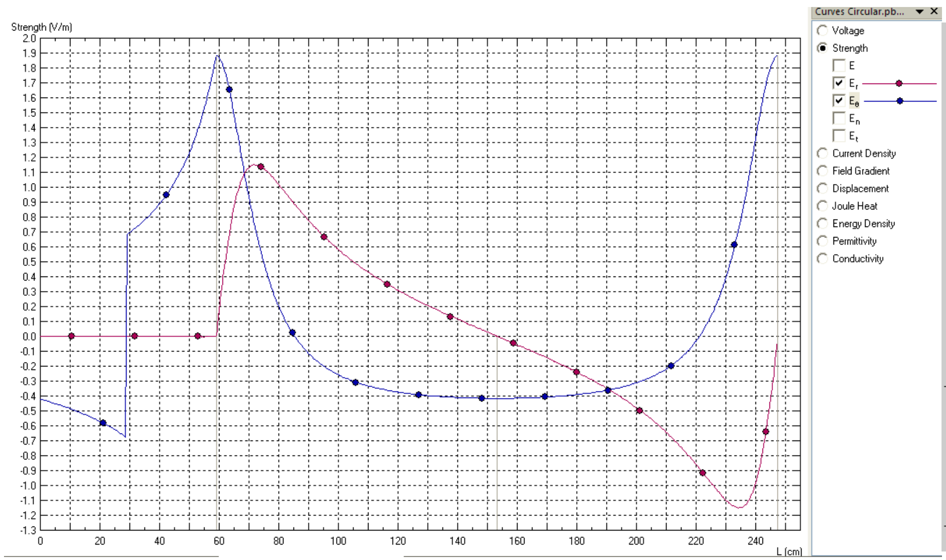
$$Q_{sup} = \frac{\varepsilon_0 V_0}{2\pi a} \tan\left(\frac{\theta}{2}\right)$$

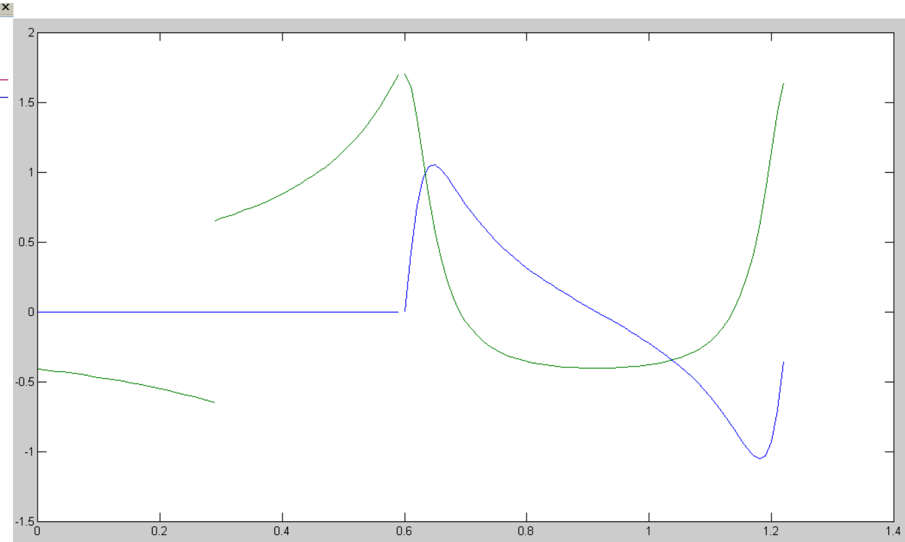
(M. A. Heald)

$$\vec{P} = \vec{E} \times \vec{H} \quad \left(W/m^2\right)$$



Campo sobre contorno interior



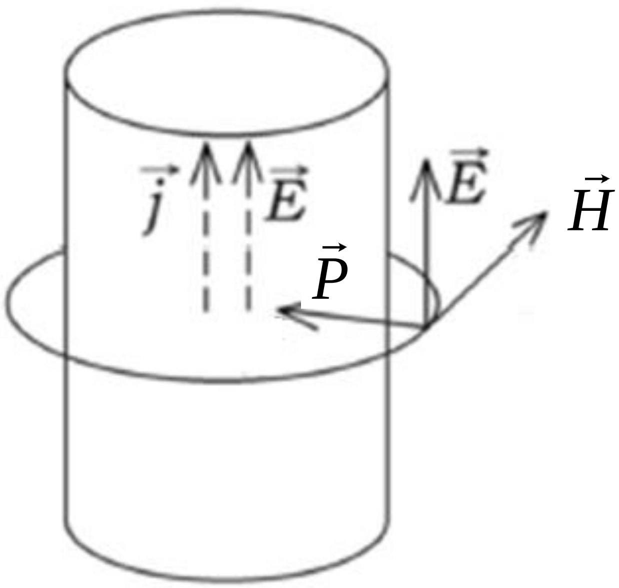


$$\vec{E}(\rho < a,\ \varphi,\ z) = -\frac{\phi_B}{\pi}\,\frac{a(\sin\varphi)\hat{\rho} + (\rho + a\cos\varphi)\hat{\varphi}}{a^2 + \rho^2 + 2a\rho\cos\varphi}$$

(Assis-Hernandes)

Representación típica del vector de Poynting fuera de un conductor que transporta una corriente eléctrica

$$\vec{P} = \vec{E} \times \vec{H} \quad \left(W/m^2\right)$$



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